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Comments on "Strictly Positive Real Transfer Functions Revisited"

H. J. Marquez and C. J. Damaren

***Abstract*—In the above paper,[1] the distinction between weak and strong strictly positive real (SPR) functions was addressed, and the feedback interconnection of a weak SPR system and a passive one was shown to be stable. The purpose of this note is to show that the proof of this lemma is actually incorrect.**

I. INTRODUCTION

The concepts of passivity and strict positive realness have been an important area of research for the last three decades. These investigations have brought a better understanding of these ideas and their applications, but also an ever increasing mismatch in the terminology adopted by different authors. The most widely accepted definitions of passivity and strict passivity are the following [1]. Define a real inner product $\langle x, y\rangle_T$ by

$$\langle x, y\rangle_T = \int_0^T x^T(t)y(t)\,dt \tag{1}$$

and let L_{2e}^n be the space of all functions $x: R^+ \to R^n$ which satisfy $\|x_T\|_2^2 = \langle x, x\rangle_T < \infty$, $\forall T \in R^+$ (R^+ is the set of positive real numbers).

Passivity: $H: L_{2e}^n \to L_{2e}^n$ is said to be passive if there exists $\beta \in R$ such that

$$\langle x, Hx\rangle_T \geq \beta, \quad \forall x \in L_{2e}^n, \quad \forall T \in R^+. \tag{2}$$

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The authors are with the Department of Engineering, Royal Roads Military College, FMO Victoria, British Columbia V0S 1B0, Canada.
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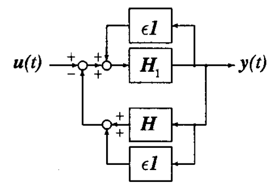

Fig. 1. The feedback system $\mathcal{S}_\epsilon$.

Strict Passivity: $H: L_{2e}^n \to L_{2e}^n$ is said to be strictly passive if there exists $\delta > 0$, and $\beta \in R$ such that

$$\langle x, Hx\rangle_T \geq \delta\|x_T\|_2 + \beta, \quad \forall x \in L_{2e}^n, \quad \forall T \in R^+. \tag{3}$$

For linear systems these definitions are closely related to the concept of strictly positive real (SPR). See the above paper[1] for the definitions of weak and strong SPR. From these definitions, it is straightforward that a linear time-invariant system whose transfer function is (weak or strong) SPR is passive but, in general, not strictly passive. For example, the system $H(s) = k/(s+a)$, $k > 0$, and $a > 0$ is SPR (and so passive); however, it is not strictly passive since $Re[H(j\omega)] \to 0$ as $\omega \to \infty$, and therefore no δ can be found to satisfy (3). The main problem with this result is that it renders the passivity theorem nearly inapplicable for linear systems, since only biproper or improper linear systems can be strictly passive.

Motivated by this observation, the authors[1] considered the class of systems which satisfy the inequality (6b)[1]

$$\frac{\displaystyle\int_0^\infty y^T u\,dt + \beta}{\displaystyle\int_0^\infty u^T u\,dt} > 0 \tag{4}$$

where $y = Hu$, and (4) is valid for all inputs u such that $\|\dot{u}\|_2/\|u\|_2 < \infty$. This class of systems was shown to be equivalent to those which are weak SPR and played a central role in Lemma 1. There, it was claimed that the feedback interconnection of a passive system and one that satisfies inequality (4) (see Fig. 1[1]) is stable. The intention of this note is to show that the proof of Lemma 1[1] is not valid. We notice here that the condition $\|\dot{u}\|_2/\|u\|_2 < \infty$ is not used in the proof of Lemma 1, and therefore in the remainder of this note it is disregarded.

Using our notation, (4) can be rewritten as

$$\frac{\langle u, Hu\rangle + \beta}{\langle u, u\rangle} > 0. \tag{5}$$

A number of comments must be made concerning this definition. In the first place, it is incomplete since for expression (4) to be well defined, it is necessary for the function u to belong to the space L_2^n. This is an important point. In fact, this issue renders incorrect the proof of Lemma 1.[1] Notice that in the proof of Lemma 1,[1] it is necessary to consider precisely the case where u is not in L_2. Yet more appropriate is to use extended spaces and rewrite (4) as

$$\lim_{T\to\infty} \frac{\langle u, Hu\rangle_T + \beta}{\langle u, u\rangle_T} > 0, \quad \forall u \in L_{2e}^n. \tag{6}$$

We now analyze the proof of Lemma 1 (Appendix II[1]). Since $u_1 = -y$ and $y_1 = u$ (refer to Fig. 1[1])

$$\frac{\langle u, Hu\rangle_T + \beta}{\langle u, u\rangle_T} + \frac{\langle u_1, H_1u_1\rangle_T + \beta_1}{\langle u, u\rangle_T} = \frac{\beta + \beta_1}{\langle u, u\rangle_T}. \tag{7}$$

Here H_1 is passive, and therefore the second term on the left-hand side of (7) is greater than or equal to zero, while H satisfies inequality (6). To show stability, the authors reason by contradiction as follows: assume that $u \notin L_2$, and take limits on both sides of (7) as $T \to \infty$. In this case, the right-hand side tends toward zero and therefore, the left-hand side also tends toward zero. Hence the authors conclude that there is a contradiction since, by (6), the left-hand side is actually greater than zero. Therefore it must be true that $u \in L_2$.

This reasoning is fallacious, however, unless condition (6) is strengthened by requiring that the following be satisfied

$$\inf_{u \in L_{2n}^n} \left[\lim_{T \to \infty} \frac{\langle u, Hu \rangle_T + \beta}{\langle u, u \rangle_T} \right] \geq \delta > 0. \qquad (8)$$

In other words, there are only two possibilities of interest in (7)

1)
$$\inf_{u \in L_{2n}^n} \left[\lim_{T \to \infty} \frac{\langle u, Hu \rangle_T + \beta}{\langle u, u \rangle_T} \right] \geq \delta > 0. \qquad (9)$$

If this is the case then indeed there is a contradiction in (7). Condition (9), however, implies that the system is strictly passive, and therefore Lemma 1 becomes a restatement of the passivity theorem (see, for example, [1]), i.e., it says nothing about weak SPR functions.

2)
$$\inf_{u \in L_{2n}^n} \left[\lim_{T \to \infty} \frac{\langle u, Hu \rangle_T + \beta}{\langle u, u \rangle_T} \right] = 0. \qquad (10)$$

In this case, there is no contradiction in (7) since the left-hand side also tends toward zero for some function u, without violating condition (6) (in the same way $1/n^p \to 0$ as $n \to \infty$, for all $p \in R^+ \geq 1$).

As a final remark we make the following observations, which emphasize the distinction between weak and strong SPR. It is relatively easy to show that the feedback combination of a (possibly nonlinear) passive plant and a strong SPR compensator is stable. The result can be proved by defining the loop transformation shown in Fig. 1 and noting that it does not alter the stability properties of the original system. It is then straightforward to show that, for small enough $\varepsilon > 0$, the system $H_1' = (1 - \varepsilon H_1)^{-1} H_1$ is passive, while $H' = H + \varepsilon 1$ is strictly passive, and therefore stability follows from the passivity theorem.

The case of a weak SPR system is, however, very different as shown in the following example.

Example 1: Consider the linear time-invariant system $H(s) = (s+c)/[(s+a)(s+b)]$, and let $H'(s) = H(s)/[1 - \epsilon H(s)]$. We have

$$H'(j\omega) + H'(-j\omega) = 2\,\frac{(abc - \epsilon c^2) + \omega^2(a + b - c - \epsilon)}{(ab - \epsilon c - \omega^2)^2 + (a + b - \epsilon)^2} > 0 \qquad (11)$$

if and only if

$$abc - \epsilon c^2 > 0, \quad a + b - c - \epsilon > 0. \qquad (12)$$

If $a + b > c$, we can always find an $\epsilon > 0$ that satisfies (12). If, however, $a + b = c$ (i.e., when $H(s)$ is weak SPR), no such $\epsilon > 0$ exists.

II. Conclusions

The proof of Lemma 1[1] is incorrect. Since this note does not disprove that the feedback interconnection of a passive plant and a weak SPR controller is stable, we conclude that it remains an open question.

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On Interval Polynomials with No Zeros in the Unit Disc

V. Blondel

Abstract—**We give a necessary condition for an interval polynomial to have no zeros in the closed unit disc. The condition is expressed in terms of the two first intervals.**

The stability analysis of polynomials subject to structured uncertainty has received considerable attention this last decade (see [2] for an historical overview; references related to this contribution include [1], [3], [5], [8], and [9]).

In this note we give a necessary condition for an interval polynomial

$$P = \{a_0 + a_1 z + \cdots + a_n z^n: \quad \underline{a}_i \leq a_i \leq \overline{a}_i\}$$

to be D-stable, i.e., such that all members of P have no roots in the closed unit disc. Our condition is expressed in terms of the two first intervals only.

In a corollary we show that if $\underline{a}_0 < \overline{a}_0/2$ and $\underline{a}_0 < \overline{a}_1/9$ then P cannot be D-stable.

The results presented here are easy consequences of a little-known theorem on analytic functions.

Landau's Theorem: Assume that the function f is analytic in the open unit disc $|z| < 1$ and that $f(z) \neq 0, 1$ for all $|z| < 1$. Then

$$|f'(0)| \leq 2|f(0)|(|\log|f(0)|| + A)$$

were A is a constant which can be taken equal to 4.4.

For a proof of this theorem (which is sometime referred to as Landau–Carathéodory theorem) see, for example, Hille [4, p. 221]. The best possible bound for A was given in 1981 by Jenkins [6]; it is equal to $4\pi^2/\Gamma^4(\frac{1}{4}) = 4.37\cdots$.

We now prove our theorem.

Theorem: Let $P = \{a_0 + a_1 z + \cdots + a_n z^n: \quad \underline{a}_i \leq a_i \leq \overline{a}_i\}$ be an interval D-stable polynomial and assume that $\overline{a}_0 > \underline{a}_0 > 0$. Then

$$|\overline{a}_1| \leq 2\underline{a}_0 \left(\log^+ \frac{\underline{a}_0}{\overline{a} - \underline{a}_0} + 4.4 \right)$$

where $\log^+ x = \max(0, \log x)$.

Proof: Define $a_0^* \in [\underline{a}_0, \overline{a}_0]$ by $a_0^* := \min(2\underline{a}_0, \overline{a}_0)$ and choose an arbitrary set of coefficients $a_i^* \in [\underline{a}_i, \overline{a}_i]$ $(i = 2, \cdots, n)$. Consider the polynomial $p(z)$ defined by

$$p(z) := \frac{1}{\underline{a}_0 - a_0^*}(\underline{a}_0 + \overline{a}_1 z + a_2^* z^2 + \cdots + a_n^* z^n).$$

It is easy to see that $p(z)$ never takes the value zero or one in the open unit disc. Indeed

$$p(z) = 0 \Leftrightarrow \underline{a}_0 + \overline{a}_1 z + a_2^* z^2 + \cdots + a_n^* z^n = 0$$

and

$$p(z) = 1 \Leftrightarrow a_0^* + \overline{a}_1 z + a_2^* z^2 + \cdots + a_n^* z^n = 0.$$

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The author is with the Division of Optimization and Systems Theory, Department of Mathematics, Royal Institute of Technology (KTH), S-100 44 Stockholm, Sweden.
IEEE Log Number 9408547.