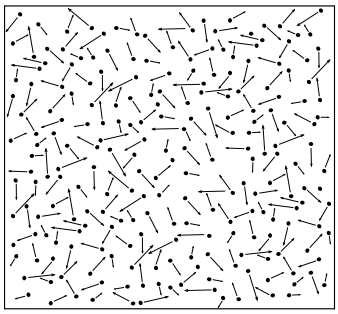
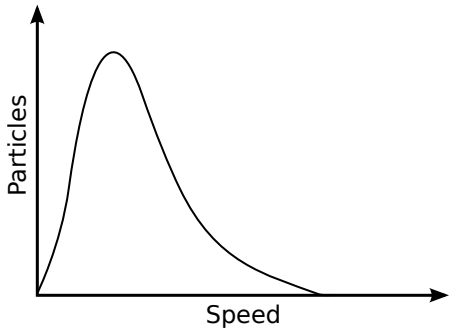
MOMENT CLOSURE METHODS FOR KINETIC EQUATIONS OF COMPLEX TRANSPORT PHENOMENA AND THEIR NUMERICAL SOLUTION

Spring 2020





0.1 Contact Information

Instructor:

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0.1 Contact Information

Teaching Assistants:

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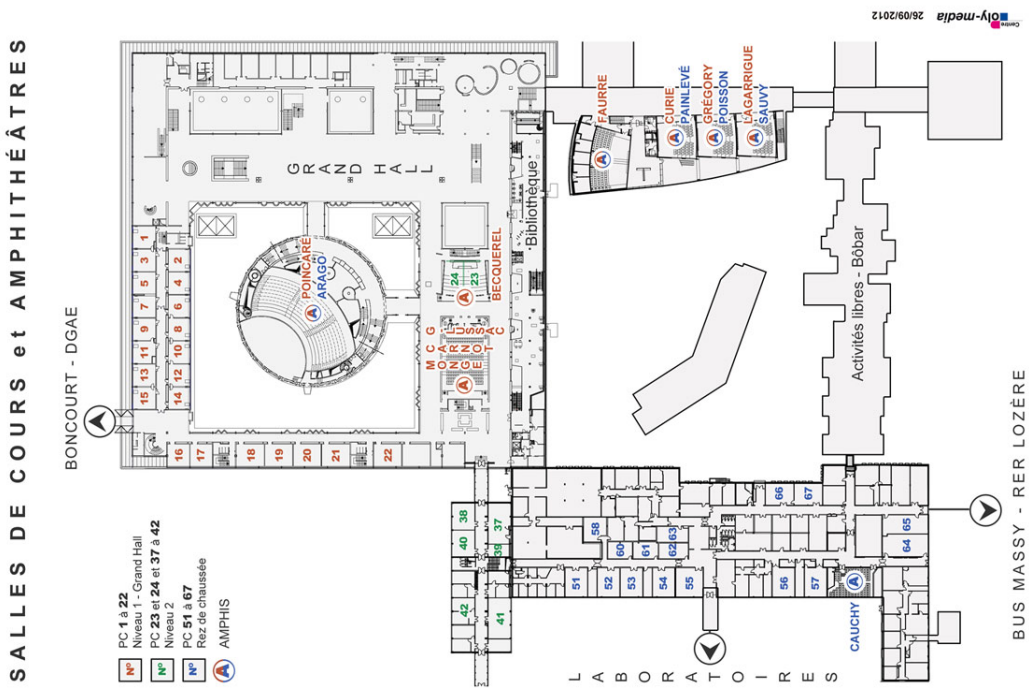
0.2 Course Information

- **Course Website**:
  `http://arrow.utias.utoronto.ca/~groth/moment-closure-course`
- **Timetable**:
  - Thursdays, 2–5 pm,
    February 27 – April 9, 2020
- **Location**:
  - Amphis Poisson, École Polytechnique

0.2.1 Location: Amphis Poisson, École Polytechnique



0.3 Gaspard Monge Visiting Professors Program

Gaspard Monge was a French mathematician, the inventor of descriptive geometry, and the father of differential geometry. During the French Revolution he served as the Minister of the Marine, and was involved in the reform of the French educational system, helping to found the École Polytechnique in 1794.

The Gaspard Monge Program is a visiting professor program supported by the Foundation of École Polytechnique. The program is aimed at internationally renowned researchers and generally senior experts or brilliant young scientists recognized internationally as leaders or future leaders in their scientific field. They reside on campus for a period of 3-4 months and are invited to give a course on a subject of their choice.

0.4 Course Outline

1. **Introduction**
   - Microscopic Versus Macroscopic Descriptions
     - Kinetic Descriptions of Transport Phenomena
     - Statistical Nature of Theories
   - Moment Closure Methods
     - Approximate Solution Method
     - Complexity Reduction via Dimensionality Reduction
   - Exemplar Kinetic Theories
     - Boltzmann Equation
     - Williams-Boltzmann Equation
     - Radiative Transfer Equation (RTE)
   - Brief History of Moment Closure Methods
   - Notation
   - Some Suggested References

0.4 Course Outline

2. **Kinetic Theory of Gases**
   - Conventional Fluid Dynamic Descriptions
     - Navier-Stokes Equations
   - Flow Regimes for a Monatomic Gas
     - Continuum, Slip, Transition, and Free-Molecular Flow Regimes
   - Statistical-Based Microscopic Description
     - Single-Component Monatomic Gas
   - Density Functions
   - Macroscopic Averages and Moments
   - Maxwell-Boltzmann Distribution
   - Boltzmann Equation
   - Boltzmann Collision Integral
     - BGK (Relaxation-Time) Approximation
   - Maxwell's Equation of Change
     - Conservation and Non-Conservation Forms
   - Boltzmann's H-Theorem
     - Equilibrium Distribution and Collisional Invariant Quantities

0.4 Course Outline

3. **Classical Method of Moments for Monatomic Gas**
   - Overview of Moment Closure Methods
   - Chapman-Enskog Method
     - Euler Equations
     - Navier-Stokes Equations
     - Burnett and Super-Burnett Equations
   - Grad's Method of Moments (Moment Closures)
     - 20-Moment Closure (Grad)
     - 13-Moment Closure (Grad)
     - 10-Moment Closure (Gaussian Closure)
     - 8-Moment Closure
     - 5-Moment Closure (Euler Equations)
   - Moment Equations
     - Quasilinear Hyperbolic System of Equations with Relaxation
     - Moment Realizability
   - Recovery of Navier-Stokes Equations
     - 13- and 20-Moment Models Contain Fluid-Limit Equations
   - Order of Magnitude Approach
   - Application of Classical Moment Methods
   - Summary of Classical Moment Methods

0.4 Course Outline

4. **Method of Moments for One-Dimensional Kinetic Theory**
   - Representative One-Dimensional Kinetic Equation
   - Chapman-Enskog Method and Fluid-Limit Solutions
     - "Euler" and "Navier-Stokes" Solutions
   - Grad's Method of Moments
     - Approximate Form for Distribution Function
     - Moment Equations and Closing Flux
     - Moment Realizability and Hyperbolicity
     - Closure Breakdown
   - Maximum-Entropy Method of Moments
     - Approximate Form for Distribution Function
     - Properties of Maximum Entropy Closures
     - Moment Equations and Closing Flux
     - Moment Realizability and Hyperbolicity
     - Closure Breakdown and Junk Subspace

0.4 Course Outline

4. **Method of Moments for One-Dimensional Kinetic Theory**
   - Quadrature-Based Method of Moments
     - QMOM and EQMOM Closures
     - Moment Equations and Closing Flux
     - Moment Realizability and Hyperbolicity
     - Closure Breakdown and Junk Subspace
   - Other Moment Approximations
   - Applications
     - Stationary Shock Waves
     - Unsteady Riemann Problems

0.4 Course Outline

5. **Maximum-Entropy Moment Closures for Monatomic Gas**
   - Hierarchy of Levermore
     - Choice of Moments
     - Positive Distribution Function
     - Symmetric Hyperbolic Systems
     - Entropy Function and Entropy Balance
     - Computational Advantages
   - Second-Moment Closures
     - Gaussian Closure
     - Application to Various Canonical Flows
   - High-Order (Beyond-Second Moment) Closures
     - Closure Breakdown and Junk Subspace
   - Interpolative-Based High-Order Maximum-Entropy Closures
     - 14-Moment Closure of McDonald and Torrilhon
     - Application to Various Canonical Flows

0.4 Course Outline

6. **Method of Moments for Polydisperse Sprays**
   - Williams-Boltzmann Equation
     - Polydisperse, Polykinetic, Multi-phase Flows and Sprays
     - Important Challenges: Particle Trajectory Crossings (PTCs); Vacuum Conditions
   - Maximum-Entropy Moment Closures
     - 1D Polykinetic Models
     - 2D Polydisperse, Polykinetic Models
   - Applications
     - PTCs
7. **Method of Moments for Radiative Heat Transfer**
   - Radiative Transfer Equation (RTE)
     - Gray and Non-Gray Participating Media
   - Spherical Harmonic Approximations (P_N Closures)
     - P_1 and P_3 Moment Closures
   - Maximum-Entropy Moment Closures (M_N Closures)
     - M_1 and M_2 Moment Closures
   - Applications
     - Gray and Non-Gray Media
     - 1D and 2D problems

0.4 Course Outline

8. **Numerical Solution of Moment Equations**
   - Quasilinear Hyperbolic System of Equations with Relaxation
     - Godonov-Type Finite-Volume Methods
     - Discontinuous-Galerkin (DG) Finite-Element Methods
   - Second-Order Godunov Finite-Volume Method
     - Piecewise Limited Linear Reconstruction
     - Riemann Solver Based Flux Functions
   - Adaptive Mesh Refinement (AMR)
     - Anisotropic Block-Based AMR