Course Description

Moment closure methods are considered for kinetic equations governing a range of complex transport phenomena, including non-equilibrium, rarefied, gaseous flow phenomena as described by the Boltzmann kinetic equation, multi-phase poly-disperse spray behaviour described by the Williams-Boltzmann equation, and radiative heat transfer in non-gray, participating media as described by the radiative transfer equation. Moment closure methods offer a means of significant complexity reduction when constructing approximate solutions to such high-dimensional equations, providing a good compromise between computational efficiency and accuracy for many practical engineering applications. Closure techniques based on classical Grad-type methods, maximum-entropy considerations, as well as quadrature formulations will all be considered. The numerical solution of the resulting systems of hyperbolic moment equations that result from the moment closure strategy will also be examined.

Syllabus

1. Introduction
   - Microscopic Versus Macroscopic Descriptions
     * Kinetic Descriptions of Transport Phenomena
     * Statistical Nature of Theories
   - Brief History of Moment Closure Methods
     * Approximate Solution Methods
     * Complexity Reduction via Dimension Reduction
   - Exemplars
     * Boltzmann Equation
     * Williams-Boltzmann Equation
     * Radiative Transfer Equation (RTE)
   - Notation

2. Kinetic Theory of Gases
   - Distribution Function and Macroscopic Averages
     * Maxwell-Boltzmann Distribution
3. **Classical Method of Moments for Monatomic Gas**

- Grad’s Method of Moments (Moment Closures)
  * 20-Moment Closure (Grad)
  * 13-Moment Closure (Grad)
  * 10-Moment Closure (Gaussian Closure)
  * 8-Moment Closure
  * 5-Moment Closure (Euler Equations)
- Moment Equations
  * Quasilinear Hyperbolic System of Equations with Relaxation
  * Moment Realizability
- Chapman-Enskog Method
  * Euler Equations
  * Navier-Stokes Equations
  * Burnett and Super-Burnett Equations
- Recovery of Navier-Stokes Equations
  * 13- and 20-Moment Models Contain Fluid-Limit Equations
- Order of Magnitude Approach

4. **Method of Moments for One-Dimensional Kinetic Theory**

- Representative One-Dimensional Kinetic Equation
- Chapman-Enskog Method and Fluid-Limit Solutions
  * “Euler” and “Navier-Stokes” Solutions
- Grad’s Method of Moments
  * Approximate Form for Distribution Function
  * Moment Equations and Closing Flux
  * Moment Realizability and Hyperbolicity
5. **Maximum-Entropy Moment Closures for Monatomic Gas**

- Hierarchy of Levermore
  * Choice of Moments
  * Positive Distribution Function
  * Symmetric Hyperbolic Systems
  * Entropy Function and Entropy Balance
  * Computational Advantages
- Second-Moment Closures
  * Gaussian Closure
  * Application to Various Canonical Flows
- High-Order (Beyond-Second Moment) Closures
  * Closure Breakdown and Junk Subspace
- Interpolative-Based High-Order Maximum-Entropy Closures
  * 14-Moment Closure of McDonald and Torrilhon
  * Application to Various Canonical Flows

6. **Method of Moments for Polydisperse Sprays**

- Williams-Boltzmann Equation
  * Polydisperse, Polykinetic, Multi-phase Flows and Sprays
  * Important Challenges: Particle Trajectory Crossings (PTCs); Vacuum Conditions
– Maximum-Entropy Moment Closures
  * 1D Polykinetic Models
  * 2D Polydisperse, Polykinetic Models
– Applications
  * PTCs

7. Method of Moments for Radiative Heat Transfer
– Radiative Transfer Equation (RTE)
  * Gray and Non-Gray Participating Media
– Spherical Harmonic Approximations (P\textsubscript{N} Closures)
  * P\textsubscript{1} and P\textsubscript{3} Moment Closures
– Maximum-Entropy Moment Closures (M\textsubscript{N} Closures)
  * M\textsubscript{1} and M\textsubscript{2} Moment Closures
– Applications
  * Gray and Non-Gray Media
  * 1D and 2D problems

8. Numerical Solution of Moment Equations
– Quasilinear Hyperbolic System of Equations with Relaxation
  * Godonov-Type Finite-Volume Methods
  * Discontinuous-Galerkin (DG) Finite-Element Methods
– Second-Order Godunov Finite-Volume Method
  * Piecewise Limited Linear Reconstruction
  * Riemann Solver Based Flux Functions
– Adaptive Mesh Refinement (AMR)
  * Anisotropic Block-Based AMR

9. Computational Examples: An additional session will be organized in which attendees are given example code and problems to explore the performance of various moment closures for 1D problems related to gaskinetic flows, disperse sprays, and radiative heat transfer.