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Engineering Notes

Geometric Approach to Spacecraft Attitude Control Using Magnetic and Mechanical Actuation

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I. Introduction

IT IS well known that spacecraft in low Earth orbit can generate control torques via the interaction of the Earth's geomagnetic field and onboard magnetic dipole moments (created via current-carrying coils) [1,2]. As mentioned in [3], the major shortcoming of magnetic actuation (as the only onboard actuator) is that control torques can only be applied to the spacecraft in a plane orthogonal to the instantaneous direction of the Earth's magnetic field, which in turn means that the spacecraft is instantaneously underactuated.

Recently, in [4,5], inertial pointing of a spacecraft using solely magnetic actuation was considered. It was shown that stabilization can be obtained while employing a quaternion and angular velocity proportional derivative (PD) type of control law. Owing to the time-varying nature of the system, the control gains are shown to be limited, which in turn leads to closed-loop performance limitations. Stability (and proof thereof) relies on averaging theory [6], which physically translates to the system possessing certain dynamic properties on average. In particular, it is assumed that on average control torques can be applied to the spacecraft in any direction owing to the fact the magnetic field is changing direction as the spacecraft orbits the Earth.

Modern spacecraft are usually endowed with magnetic torquers and some type of mechanical actuator, such as reaction wheels. The magnetic torquers are usually used for detumbling of the spacecraft upon egress from the launch vehicle, as well as momentum dumping of reaction wheels. Reaction wheels are used for fine attitude control. Seldom are both magnetic torquers and reaction wheels intended to work together harmoniously in concert. Having both actuation systems work simultaneously can lead to power savings (depending on, among other things, orbit inclination, control scheme and gains, etc. [7]), as well as reduce reaction wheel torque requirements. Additionally, although most spacecraft are equipped with redundant reaction or momentum wheels, failure of both primary and secondary wheels in one axis is possible, as discussed in [8]. Upon the failure of primary and redundant pitch axis wheels, the attitude control system of RADARSAT-1 was redesigned (and subsequently uploaded while on orbit) to use the remaining wheels and magnetic actuation together, thus saving the mission. Attitude control of spacecraft using two actuation systems was also considered in [9,10]. Motivated by [4,5], in [9] the same magnetic control law was augmented with reaction wheels; sufficient conditions were given such that the gain limited nature of the magnetic control law was relaxed, leading to better closed-loop system performance. In [10] the attitude control of a spacecraft using both magnetic torquers and thrusters based on a linear time-periodic model was considered, leading to linear time-invariant and linear time-periodic control designs. Actuator saturation was also considered.

In this paper we consider the control of a spacecraft using both magnetic and mechanical actuation in tandem. We present a geometric scheme whereby the control vector is decomposed into orthogonal and parallel components with respect to the orientation of the instantaneous magnetic field vector. The spacecraft magnetic torquers apply the orthogonal control component, while the remaining parallel component is applied by mechanical actuators, specifically, reaction wheels. We show that our control decomposition is not limited to spacecraft equipped with three wheels, but those equipped with one, two, or three wheels. Additionally, saturation of the torque rods is considered. The effectiveness of our method is shown to work well in simulation while employing an adaptive tracking controller.

II. Spacecraft Attitude Dynamics and Actuation

We wish to control the attitude of a generic rigid-body spacecraft in low Earth orbit. The rotational dynamics are governed by Euler's equation [11]:

$$\mathbf{I}\,\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}\mathbf{I}\boldsymbol{\omega} = \mathbf{u} = \boldsymbol{\tau}_m + \boldsymbol{\tau}_w \tag{1}$$

where $\mathbf{I}$ is the moment of inertia matrix, $\boldsymbol{\omega}$ is the angular velocity of the spacecraft expressed in the body-fixed frame, and

$$\mathbf{a}^{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

is a skew-symmetric matrix satisfying $\mathbf{a}^{\times T} = -\mathbf{a}^{\times}$, where $\mathbf{a} = [a_1 \quad a_2 \quad a_3]^T$. The control torque, $\mathbf{u}$, will subsequently be distributed between wheel torques, $\boldsymbol{\tau}_w$, and magnetic torques, $\boldsymbol{\tau}_m(t) = \mathbf{b}^{\times T}(t)\mathbf{m}(t)$, where $\mathbf{b}$ is the Earth's geomagnetic field vector expressed in the body-fixed frame and $\mathbf{m}$ is the magnetic dipole moment. The vector $\mathbf{b}$ is not constant for two reasons: both the spacecraft attitude and position are changing while on orbit.

The attitude of a spacecraft can be described by the four parameter quaternion set $\boldsymbol{\epsilon} = [\epsilon_1 \quad \epsilon_2 \quad \epsilon_3]^T$ and η, which together satisfy $\boldsymbol{\epsilon}^T\boldsymbol{\epsilon} + \eta^2 = 1$ [11]. The quaternion rates and the angular velocity are related by

$$\begin{bmatrix} \dot{\boldsymbol{\epsilon}} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \eta\mathbf{1} + \boldsymbol{\epsilon}^{\times} \\ -\boldsymbol{\epsilon}^T \end{bmatrix}\boldsymbol{\omega} \quad \text{or} \quad \boldsymbol{\omega} = 2(\eta\dot{\boldsymbol{\epsilon}} - \dot{\eta}\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{\times}\dot{\boldsymbol{\epsilon}}) \tag{2}$$

III. Geometric Decomposition of Control

Consider a spacecraft in low Earth orbit equipped with magnetic torquers and reactions wheels governed by Eqs. (1) and (2) and a control system that provides the control torque, $\mathbf{u}$, to be applied to the spacecraft. The control system is not assumed to be designed using any one theory; for example, the control system could be a simple PD control law, or designed via linear quadratic regulator theory, optimal periodic control theory, $\mathcal{H}_\infty$ theory, etc. [3].

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A. Control Decomposition with Three Reaction Wheels Available

Nominally, the spacecraft to be controlled is equipped with three orthogonal torque rods and three orthogonal reaction wheels. As mentioned in the Introduction, a general control signal cannot be fully realized via magnetic actuation alone because the magnetic torquers can only apply torques to the spacecraft in a plane orthogonal to the instantaneous **b** field; any **m** that is parallel to **b** results in zero torque.

Consider a general control signal **u** decomposed into parallel and perpendicular components with respect to the Earth's magnetic field vector expressed in the spacecraft body frame:

$$\mathbf{u} = \mathbf{u}_\perp + \mathbf{u}_\parallel \tag{3}$$

where $\mathbf{u}_\perp \in \mathrm{Im}\{\mathbf{b}^\times\}$ and $\mathbf{u}_\parallel \in \mathrm{Ker}\{\mathbf{b}^\times\}$, as shown in Fig. 1. This "natural" decomposition will be used to distribute the control between the magnetic torquers and the reaction wheels. Logically, it follows that

$$\boldsymbol{\tau}_m = \mathbf{u}_\perp, \qquad \boldsymbol{\tau}_w = \mathbf{u}_\parallel \tag{4}$$

because the control that lies in $\mathrm{Ker}\{\mathbf{b}^\times\}$, that is $\mathbf{u}_\parallel$, cannot be applied to the system through the magnetic torquers. Thus, the control that does not lie in the Kernel (that is, what lies in the Image, $\mathbf{u}_\perp$) is applied via magnetic actuation and the "left over control" $\mathbf{u}_\parallel$ is applied by the reaction wheels.

It is straightforward to show using linear algebra that

$$\mathbf{u}_\perp = \hat{\mathbf{b}}^{\times^T}\hat{\mathbf{b}}^\times\mathbf{u}, \qquad \mathbf{u}_\parallel = \hat{\mathbf{b}}\hat{\mathbf{b}}^T\mathbf{u} \tag{5}$$

where $\hat{(\cdot)}$ denotes a unit vector (i.e., $\hat{\mathbf{b}} = \|\mathbf{b}\|^{-1}\mathbf{b}$). Via the above geometric decomposition, the control signal **u** is applied completely, which can be seen by adding $\boldsymbol{\tau}_m$ and $\boldsymbol{\tau}_w$ while using Eq. (5) and the identity $\hat{\mathbf{b}}^{\times^T}\hat{\mathbf{b}}^\times = \mathbf{1} - \hat{\mathbf{b}}\hat{\mathbf{b}}^T$. To apply $\boldsymbol{\tau}_m$ to the spacecraft body, we must find **m** such that $\boldsymbol{\tau}_m = \mathbf{b}^{\times T}\mathbf{m}$. From Eqs. (4) and (5),

$$\mathbf{b}^{\times T}\mathbf{m} = \hat{\mathbf{b}}^{\times^T}\hat{\mathbf{b}}^\times\mathbf{u} \tag{6}$$

which must be solved for **m**. Because $\boldsymbol{\tau}_m \in \mathrm{Im}\{\mathbf{b}^\times\}$ and $\mathbf{u}_\perp \in \mathrm{Im}\{\mathbf{b}^\times\}$, a solution, **m**, to Eq. (6) is guaranteed to exist, although the solution is not unique because $\mathbf{b}^{\times T}$ is not invertible. Finding **m** such that $\boldsymbol{\tau}_m$ is applied to the spacecraft will be discussed in Sec. IV.

B. Control Decomposition with Less Than Three Reaction Wheels Available

In the previous section we used a geometric decomposition to distribute the control between magnetic torquers and reaction wheels. With three wheels present, the wheel torques can exactly equal $\mathbf{u}_\parallel$. Let us now consider the case in which only one or two wheels are available, either by design (to, for example, save weight and minimize design complexity) or as a result of wheel failure during mission operation.

Consider again a general control signal and the Earth's magnetic field vector, but also magnetic torques and geometrically limited wheel torques to be applied to the spacecraft as shown in Fig. 2. The wheels are able to apply torques about some general direction **a**, which is appropriately restricted according to the particular wheels available. For instance, if two wheels are available, **a** may be any unit vector in the plane defined by the spin axes of the wheels.

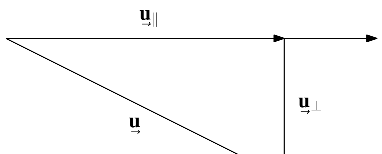

Fig. 1 Geometric decomposition of control vector with respect to b field.

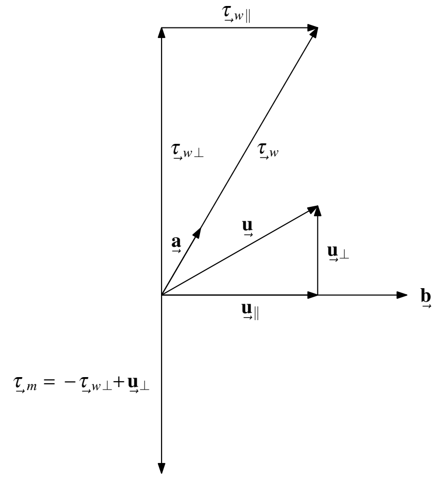

Fig. 2 Decomposition of u into magnetic torques and wheel torques (less than three wheels available).

Alternatively, if only one wheel is available, **a** is a fixed unit vector coinciding with the single wheel spin axis.

Given a reduced number of wheels, only a component of the wheel torques, that being $\boldsymbol{\tau}_{w\parallel}$, lies in $\mathrm{Ker}\{\mathbf{b}^\times\}$. It follows that $\boldsymbol{\tau}_m$ and $\boldsymbol{\tau}_w$ must sum together to create **u**:

$$\mathbf{u} = \boldsymbol{\tau}_m + \boldsymbol{\tau}_w \tag{7}$$

where

$$\boldsymbol{\tau}_m \in \mathrm{Im}\{\mathbf{b}^\times\}, \qquad \boldsymbol{\tau}_{w\parallel} \in \mathrm{Ker}\{\mathbf{b}^\times\}$$

Letting $\boldsymbol{\tau}_m = -\boldsymbol{\tau}_{w\perp} + \mathbf{u}_\perp$ (as shown in Fig. 2) and adding $\boldsymbol{\tau}_w$ creates the desired control:

$$\mathbf{u} = \boldsymbol{\tau}_m + \boldsymbol{\tau}_w = -\boldsymbol{\tau}_{w\perp} + \mathbf{u}_\perp + \boldsymbol{\tau}_w = \mathbf{u}_\perp + \boldsymbol{\tau}_{w\parallel} = \mathbf{u}_\perp + \mathbf{u}_\parallel$$

where $\mathbf{u}_\parallel \equiv \boldsymbol{\tau}_{w\parallel}$.

To find $\boldsymbol{\tau}_w$ we must use the relation

$$\mathbf{u}_\parallel = \boldsymbol{\tau}_{w\parallel} \Leftrightarrow \hat{\mathbf{b}}\hat{\mathbf{b}}^T\mathbf{u} = \hat{\mathbf{b}}\hat{\mathbf{b}}^T\boldsymbol{\tau}_w \tag{8}$$

which in turn depends on the number of wheels present. Once $\boldsymbol{\tau}_w$ is known, $\boldsymbol{\tau}_m$ can be found.

1. Two Reaction Wheels Available

Assume there are two reaction wheels available, each aligned with the $\mathbf{1}_2$ and $\mathbf{1}_3$ axes of the spacecraft such that

$$\boldsymbol{\tau}_w = [\mathbf{1}_2 \quad \mathbf{1}_3]\boldsymbol{\tau}_{w2a}$$

where $\mathbf{1}_2 = [0 \quad 1 \quad 0]^T$, $\mathbf{1}_3 = [0 \quad 0 \quad 1]^T$, and $\boldsymbol{\tau}_{w2a} = [\tau_2 \quad \tau_3]^T$. The wheels could be aligned with any other set of axes, but we choose this particular set for simplicity of exposition. We seek to find $\boldsymbol{\tau}_w$. From Eq. (8) we can write

$$\begin{bmatrix} b_1(\mathbf{b}^T\mathbf{u}) \\ b_2(\mathbf{b}^T\mathbf{u}) \\ b_3(\mathbf{b}^T\mathbf{u}) \end{bmatrix} = \begin{bmatrix} b_1b_2 & b_1b_3 \\ b_2^2 & b_2b_3 \\ b_2b_3 & b_3^2 \end{bmatrix}\begin{bmatrix} \tau_2 \\ \tau_3 \end{bmatrix}$$

$$\Leftrightarrow \mathbf{b}^T\mathbf{u} = \mathbf{b}^T[\mathbf{1}_2 \quad \mathbf{1}_3]\boldsymbol{\tau}_{w2a}, \quad \forall\ b_2, \qquad b_3 \neq 0$$

Provided **b** is not perpendicular to the $\mathbf{1}_2$–$\mathbf{1}_3$ plane, an infinite number of solutions to the above relation exist, owing to the fact that the equation to be solved is underdetermined.

In the interest of picking τ_2 and τ_3 optimally, we will express τ_3 in terms of τ_2 ($\tau_3 = 1/b_3(\mathbf{b}^T\mathbf{u} - b_2\tau_2)$) and pick τ_2 to be the solution of the following minimization problem: minimize $\mathcal{J}_\tau(\tau_2) = \frac{1}{2}(\tau_2^2 + \tau_3^2)$. Setting $d\mathcal{J}_\tau/d\tau_2 = 0$ leads to $\tau_2 = (b_2/(b_2^2 + b_3^2))\mathbf{b}^T\mathbf{u}$ and

$\tau_3 = (b_3/(b_2^2 + b_3^2))\mathbf{b}^T\mathbf{u}$. Note, provided $\mathbf{b} \notin \mathrm{Im}\{\mathbf{1}_1\}$ $b_2 \neq 0$ and $b_3 \neq 0$.

Having found $\boldsymbol{\tau}_w$, we can now calculate $\boldsymbol{\tau}_m$:

$$\boldsymbol{\tau}_m = -\boldsymbol{\tau}_{w\perp} + \mathbf{u}_\perp = \hat{\mathbf{b}}^{\times^T}\hat{\mathbf{b}}^\times(-\boldsymbol{\tau}_w + \mathbf{u}) \tag{9}$$

2. One Reaction Wheel Available

Assuming there is only one reaction wheel spinning about the $\mathbf{1}_3$ axis of the spacecraft, it follows that

$$\boldsymbol{\tau}_w = \mathbf{1}_3\tau_3$$

Again, using Eq. (8) we have

$$\tau_3 = \frac{\mathbf{b}^T\mathbf{u}}{b_3} \Leftrightarrow \boldsymbol{\tau}_w = \mathbf{1}_3\frac{\mathbf{b}^T\mathbf{u}}{b_3}, \quad \forall\ b_3 \neq 0$$

Provided $\mathbf{b}$ does not lie in the $\mathbf{1}_1$–$\mathbf{1}_2$ plane $b_3 \neq 0$. Knowing $\boldsymbol{\tau}_w$, $\boldsymbol{\tau}_m$ can be determined via Eq. (9). Note that we have chosen the $\mathbf{1}_3$ axis and τ_3 arbitrarily; we could have picked the $\mathbf{1}_2$ axis and τ_2, or the $\mathbf{1}_1$ axis and τ_1, and solved for τ_2 and τ_1, respectively.

IV. Calculation of Magnetic Dipole Moment

From Eqs. (4) and (5) we have $\boldsymbol{\tau}_m = \hat{\mathbf{b}}^{\times^T}\hat{\mathbf{b}}^\times\mathbf{u}$. To apply this torque to the spacecraft, we must determine the magnetic dipole moment $\mathbf{m}$ such that $\mathbf{b}^{\times T}\mathbf{m} = \boldsymbol{\tau}_m$. Writing out $\mathbf{b}^{\times T}\mathbf{m} = \boldsymbol{\tau}_m$, we have

$$-\begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} \tau_{m1} \\ \tau_{m2} \\ \tau_{m3} \end{bmatrix} \Leftrightarrow \begin{array}{l} m_1 = \frac{1}{b_3}(-\tau_{m2} + m_3 b_1) \\ m_2 = \frac{1}{b_3}(\tau_{m1} + m_3 b_2) \end{array}$$

where $\boldsymbol{\tau}_m = [\tau_{m1} \quad \tau_{m2} \quad \tau_{m3}]^T$. Dipoles m_1 and m_2 are expressed as a function of m_3; thus, there are an infinite number of possible $\mathbf{m}$ values that satisfy $\mathbf{b}^{\times T}\mathbf{m} = \boldsymbol{\tau}_m$ (provided $b_3 \neq 0$).

Although we can pick any m_3 value, we wish to pick a set of m_j that are optimal (for $j = 1, 2, 3$) in some sense. In the interest of minimizing power, let us solve the following simple optimization problem: minimize $\mathcal{J}_m(m_3) = \frac{1}{2}\mathbf{m}^T\mathbf{m} = \frac{1}{2}(m_1^2 + m_2^2 + m_3^2)$. Letting $d\mathcal{J}_m/dm_3 = 0$ yields

$$\mathbf{m} = \begin{bmatrix} \frac{1}{b_3}(-\tau_{m2} + m_3 b_1) \\ \frac{1}{b_3}(\tau_{m1} + m_3 b_2) \\ \frac{1}{\mathbf{b}^T\mathbf{b}}(\tau_{m2}b_1 - \tau_{m,1}b_2) \end{bmatrix}, \quad \forall\ b_3 \neq 0 \Leftrightarrow \mathbf{m} = \frac{\mathbf{b}^\times\boldsymbol{\tau}_m}{\mathbf{b}^T\mathbf{b}} \tag{10}$$

where we have used the fact $\mathbf{b}^T\boldsymbol{\tau}_m = 0$. Note the second expression for $\mathbf{m}$ in Eq. (10) does not suffer from division by b_3.

V. Geometric Decomposition of Control Considering Magnetic Dipole Saturation

In a practical context, current limitations associated with the magnetic torque rods impose a saturation limit on $\mathbf{m}$ such that $|m_j| \leq m_\Box$, where $m_\Box$ is the saturation limit of each identical torque rod. We will now extend our geometric method to handle saturation issues associated with magnetic actuation.

A. Three Reaction Wheels Available

Consider the desired control decomposed into parallel and perpendicular components, but also into a wheel torque vector, $\boldsymbol{\tau}_w$, and a saturated magnetic torque vector, $\boldsymbol{\tau}_m^s$:

$$\mathbf{u} = \mathbf{u}_\parallel + \mathbf{u}_\perp = \boldsymbol{\tau}_w + \boldsymbol{\tau}_m^s \tag{11}$$

where $\boldsymbol{\tau}_m^s = k_m^s\hat{\boldsymbol{\tau}}_m$, $\hat{\boldsymbol{\tau}}_m = \mathbf{b}^{\times T}\mathbf{m}'$ is the magnetic torque unit vector, $\mathbf{m}' = \|\boldsymbol{\tau}_m\|^{-1}\mathbf{m}$ is computed using the procedure outlined in Sec. IV, and $k_m^s = \min(|\frac{m_\Box}{m_1'}|, |\frac{m_\Box}{m_2'}|, |\frac{m_\Box}{m_3'}|)$ is the saturated magnetic gain. Starting with the first part of the expression in Eq. (11), by adding and subtracting $\boldsymbol{\tau}_m^s$ we can write

$$\mathbf{u} = \underbrace{\mathbf{u}_\parallel + \mathbf{u}_\perp - \boldsymbol{\tau}_m^s}_{\tau_w} + \boldsymbol{\tau}_m^s$$

Therefore, $\boldsymbol{\tau}_w = \mathbf{u}_\parallel + \mathbf{u}_\perp - \boldsymbol{\tau}_m^s$. The wheel torques compensate for the lack of magnetic actuation as a result of saturation by applying a torque that lies primarily in $\mathrm{Ker}\{\mathbf{b}^\times\}$, but also has a small component in $\mathrm{Im}\{\mathbf{b}^\times\}$.

B. Less Than Three Reaction Wheels Available

Consider a spacecraft equipped with magnetic actuators that may saturate, as well as one or two wheels. Such a sparse actuator configuration disallows the desired control, $\mathbf{u}$, to be fully realized. However, a control torque that is reduced in magnitude as a result of saturation, $\mathbf{u}^s$, is permissible. Given that $\mathbf{u}^s$ can be applied, it follows that the direction of each control vector be equivalent, that is, $\hat{\mathbf{u}}^T\hat{\mathbf{u}}^s = 1$. Ideally, the control system should be designed with saturation accounted for, thus guaranteeing closed-loop stability (see, for example, [12]), but in the case of sudden wheel failure a practical working solution may be needed quickly, and as such the following method is proposed. Unfortunately, at present we cannot guarantee closed-loop stability, although the method works well in simulation, as will be shown in Sec. VI.

Ignoring saturation for a moment, we can write

$$\mathbf{u} = k_{ma}\hat{\boldsymbol{\tau}}_m + k_{wa}\hat{\boldsymbol{\tau}}_w$$

where

$$k_{ma} = \frac{(\hat{\boldsymbol{\tau}}_m^T - (\hat{\boldsymbol{\tau}}_w^T\hat{\boldsymbol{\tau}}_m)\hat{\boldsymbol{\tau}}_w^T)\mathbf{u}}{1 - (\hat{\boldsymbol{\tau}}_w^T\hat{\boldsymbol{\tau}}_m)^2} \quad \text{and} \quad k_{wa} = \frac{(\hat{\boldsymbol{\tau}}_w^T - (\hat{\boldsymbol{\tau}}_m^T\hat{\boldsymbol{\tau}}_w)\hat{\boldsymbol{\tau}}_m^T)\mathbf{u}}{1 - (\hat{\boldsymbol{\tau}}_m^T\hat{\boldsymbol{\tau}}_w)^2}$$

are the magnetic gain and reaction wheel gain, respectively, when the spacecraft is equipped with less than three wheels, and $\hat{\boldsymbol{\tau}}_m$ and $\hat{\boldsymbol{\tau}}_w$ are the unit vectors associated with the magnetic torquers and reaction wheel(s). Thus, while disallowing the possibility of saturation $\mathbf{u} = \boldsymbol{\tau}_m + \boldsymbol{\tau}_w$, where $\boldsymbol{\tau}_m = k_{ma}\hat{\boldsymbol{\tau}}_m$ and $\boldsymbol{\tau}_w = k_{wa}\hat{\boldsymbol{\tau}}_w$, which is equivalent to Eq. (7). Upon saturation of any one of the torque rods, the total magnetic torque to be applied to the spacecraft must scale accordingly. Similar to Sec. V.A, $\boldsymbol{\tau}_m^s = k_{ma}^s\hat{\boldsymbol{\tau}}_m = \mathbf{b}^{\times T}\mathbf{m}^s$, where $\mathbf{m}^s = k_{ma}^s\mathbf{m}'$ and $k_{ma}^s = \min(|\frac{m_\Box}{m_1'}|, |\frac{m_\Box}{m_2'}|, |\frac{m_\Box}{m_3'}|)$. Both $\boldsymbol{\tau}_m$ and $\boldsymbol{\tau}_m^s$ point in the same direction, but have different magnitudes. It follows that

$$\mathbf{u}^s = k_{ma}^s\hat{\boldsymbol{\tau}}_m + k_{wa}^s\hat{\boldsymbol{\tau}}_w$$

where $k_{wa}^s = \frac{k_{ma}^s k_{wa}}{k_{ma}}$ owing to the requirement that $\hat{\mathbf{u}}^T\hat{\mathbf{u}}^s = 1$. It follows that the torques to be applied by the magnetic actuators and reaction wheel(s) are $\boldsymbol{\tau}_m^s = k_{ma}^s\hat{\boldsymbol{\tau}}_m$ and $\boldsymbol{\tau}_w^s = k_{wa}^s\hat{\boldsymbol{\tau}}_w$, respectively. Note the magnitude of the wheel torques scale so that $\hat{\mathbf{u}}$ and $\hat{\mathbf{u}}^s$ are collinear.

VI. Numerical Example

Consider a spacecraft with $\mathbf{I} = \mathrm{diag}\{27, 17, 25\}\ \mathrm{kg \cdot m^2}$ in a Keplerian orbit at an altitude of 450 km with zero eccentricity, an inclination of 87 deg, the angle of the right ascension of the ascending node equal to zero, the argument of perigee equal to zero, and the time of perigee passage equal to zero. In simulation, we will use the magnetic field model described in Wertz [1], Appendix H, and restated in [9].

In [13], an adaptive control scheme for attitude tracking control of a spacecraft is derived, which we will implement here in conjunction with our geometric approach to distribute torques between magnetic torquers and reaction wheels. Consider the following desired trajectory:

$$\boldsymbol{\epsilon}_d = \mathbf{1}_3\sin(f/2), \qquad \eta_d = \cos(f/2)$$

where f is the true anomaly of the spacecraft orbit. Using Eq. (2) and $\boldsymbol{\epsilon}_d$, η_d as described above, the desired angular velocity, $\boldsymbol{\omega}_d$, can be derived. The attitude error is

$$\begin{bmatrix} \boldsymbol{\epsilon}_e \\ \eta_e \end{bmatrix} = \begin{bmatrix} \eta_d \mathbf{1} - \boldsymbol{\epsilon}_d^\times & -\boldsymbol{\epsilon}_d \\ \boldsymbol{\epsilon}_d^T & \eta_d \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon} \\ \eta \end{bmatrix}$$

Let $\boldsymbol{\omega}_r = \boldsymbol{\omega}_d - \lambda\boldsymbol{\epsilon}_e$, $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{\omega}_d$, and $\boldsymbol{\rho} = \boldsymbol{\omega}_e + \lambda\boldsymbol{\epsilon}_e$, where $\lambda > 0$. Consider the following control:

$$\mathbf{u} = \mathbf{W}(\boldsymbol{\omega}_r, \dot{\boldsymbol{\omega}}_r, \boldsymbol{\omega})\hat{\boldsymbol{\alpha}} - \mathbf{K}\boldsymbol{\rho} \tag{12}$$

which is composed of a feedforward component, $\mathbf{W}(\boldsymbol{\omega}_r, \dot{\boldsymbol{\omega}}_r, \boldsymbol{\omega})\hat{\boldsymbol{\alpha}}$, and a feedback component, $-\mathbf{K}\boldsymbol{\rho}$, where $\mathbf{K} > 0$ is the feedback gain matrix. The matrix $\mathbf{W}$ is the regressor matrix defined so that

$$\mathbf{I}\,\dot{\boldsymbol{\omega}}_r + \boldsymbol{\omega}_r^\times \mathbf{I}\boldsymbol{\omega} = \mathbf{W}(\boldsymbol{\omega}_r, \dot{\boldsymbol{\omega}}_r, \boldsymbol{\omega})\boldsymbol{\alpha}$$

while $\boldsymbol{\alpha}$ are the spacecraft inertia matrix elements and $\hat{\boldsymbol{\alpha}}$ are their estimates. The error between the actual and estimated inertia, $\tilde{\boldsymbol{\alpha}} = \boldsymbol{\alpha} - \hat{\boldsymbol{\alpha}}$, is governed by

$$\dot{\tilde{\boldsymbol{\alpha}}} = \Gamma^{-1}\mathbf{W}^T\boldsymbol{\rho}, \qquad \dot{\hat{\boldsymbol{\alpha}}} = -\Gamma^{-1}\mathbf{W}^T\boldsymbol{\rho}$$

where $\Gamma > 0$ is the adaptive gain matrix. It can be shown that using the control presented in Eq. (12) ensures that $\boldsymbol{\omega}$, $\boldsymbol{\epsilon}$, and η asymptotically track $\boldsymbol{\omega}_d$, $\boldsymbol{\epsilon}_d$, and η_d.

Values of $\lambda = 0.0075$ rad/s, $\mathbf{K} = (0.075)\mathbf{1}$ kg $\cdot$ m^2/s, and $\Gamma^{-1} = (1/15)\mathbf{1}$ kg $\cdot$ m$^2 \cdot$ s^2 will be used in simulation, where $\mathbf{1}$ is the identity matrix. The initial conditions are $\boldsymbol{\epsilon} = \mathbf{0}$, $\eta = 1$, and $\omega_1 = \omega_2 = \omega_3 = 0.02$ rad/s. We will consider a spacecraft equipped with three current limited magnetic torquers and one reaction wheel aligned with the $\mathbf{1}_3$ axis of the spacecraft. The saturation limit of the torque rods is $m_\square = 25$ A $\cdot$ m^2. The control is decomposed using the procedures presented in Secs. III.B.2 and V.B. Figure 3 shows the angular velocity and quaternion evolution of the spacecraft versus orbit, whereas Fig. 4 shows the magnetic torques, the reaction wheel torque, and the magnetic dipole moments versus orbit.

VII. Remarks

The geometric control decomposition developed in this paper has several advantages compared to other methods. To start, the method presented in [9] distributes the magnetic torques and wheel torques independently; the two control laws operate without any knowledge of each other. The magnetic torques and wheel torques could be overlapping and, in effect, negate each other over a short period of time. Our geometric method ensures that the magnetic torques and wheel torques do not overlap due to the fact each torque lies in a unique subspace of $\mathbf{b}^\times$.

In [10], two different methods designed to account for saturation of the actuators are presented that use numerical optimization algorithms. Having numerical optimization algorithms solve for a set of admissible torques each time a new control is computed is extremely demanding computationally, and possibly infeasible while on orbit. Our method decomposes the control geometrically, which is mathematically rigorous, and any subsequent optimization (for example, calculation of wheel torques or the magnetic dipole

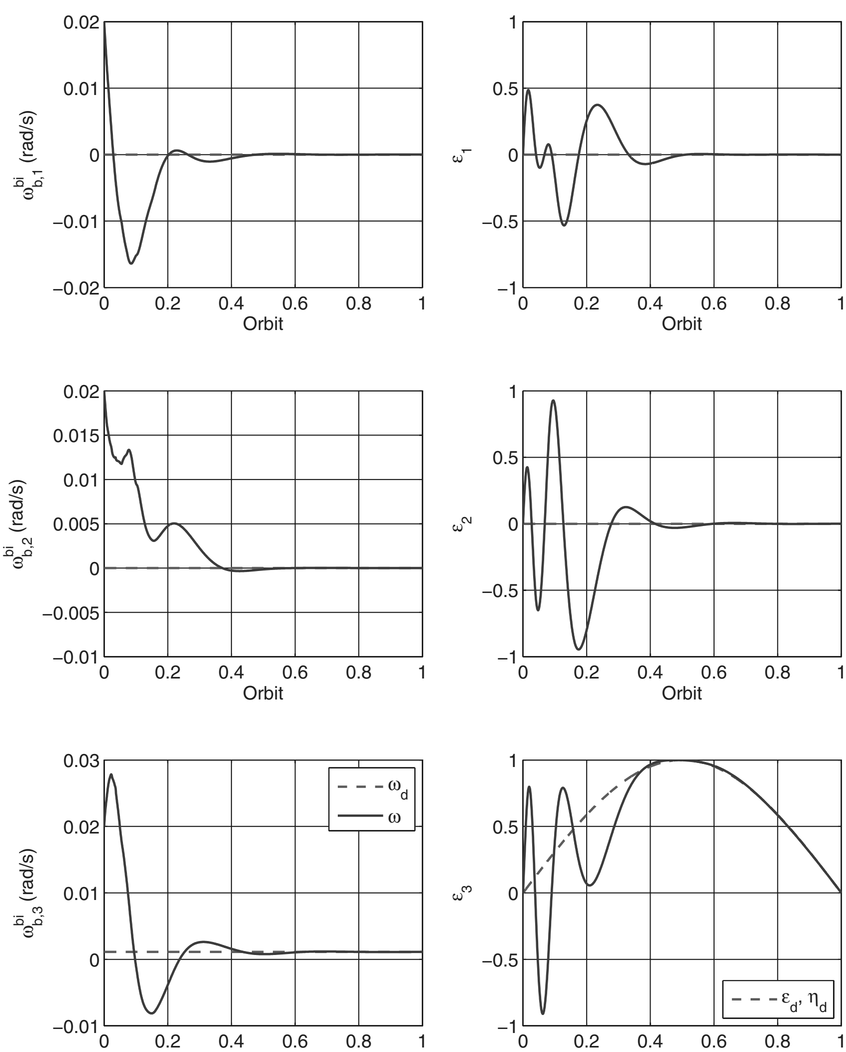

Fig. 3 Angular velocity and quaternions versus orbit.

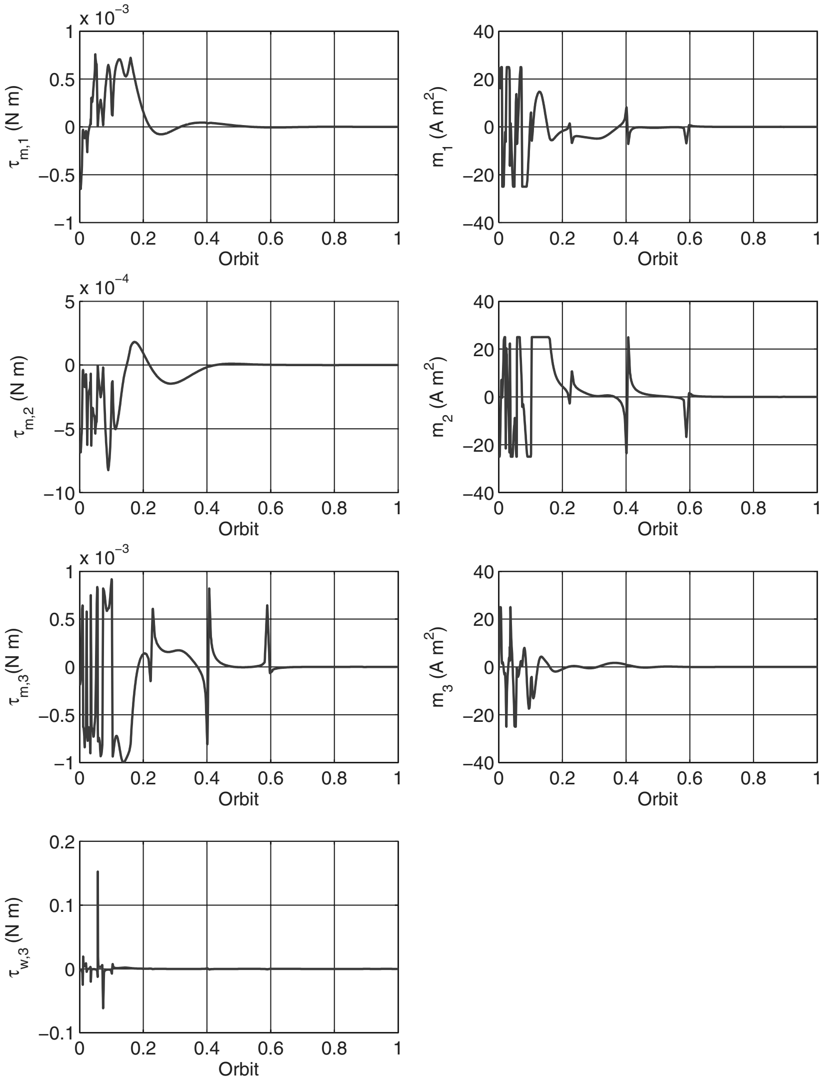

Fig. 4 Magnetic torque, wheel torque, and magnetic dipole moment versus orbit.

moment) is analytical, leading to exact equations. Additionally, we handle saturation constraints via geometric decomposition, again leading to exact equations. Our method is, from a practical point of view, computationally tractable.

Although previous papers consider a combination of magnetic and mechanical actuation (in the form of reaction wheels and thrusters), they do not discuss the possibility of having less than three wheels. Our geometric method can be used for systems with one, two, or three orthogonal reaction wheels.

Finally, what is attractive about the geometric control decomposition presented is that it can be used in conjunction with any control scheme. We have chosen to implement an adaptive scheme in simulation, but any other control theory/method can be used.

Although our geometric scheme has many advantages, there is a perceived disadvantage related to instantaneous underactuation when the spacecraft is equipped with less than three wheels. Recall from Sec. III.B.1 that wheel torques were derived assuming that the **b** field was oriented such that $b_2 \neq 0$ and $b_3 \neq 0$ (or, with respect to Sec. III.B.2, such that $b_3 \neq 0$). First, our scheme should logically be used in conjunction with orbits and desired spacecraft attitudes that avoid a **b** field yielding $b_2 = b_3 = 0$. Second, although it is true that the wheel torques cannot be solved for when $b_2 = b_3 = 0$, if this were the case it would be only for a short duration, as the spacecraft would either change orientation or change position relative to the Earth; hence, the **b** field would change and nonzero b_2 and b_3 values would ensue. We are essentially arguing, much like the argument presented in [4,5], that underactuation in a particular direction is instantaneous. Therefore, the issue of underactuation given one or two wheels is not major.

VIII. Conclusions

This paper considers the attitude control of spacecraft in low Earth orbit using both magnetic and mechanical actuation in tandem. We have presented a method by which the control signal is decomposed into two parts, with the component of the control lying orthogonal to the **b** field being applied via magnetic actuation and the remainder of the control (that is, what lies parallel to the **b** field) being applied via mechanical actuation. Saturation of the actuators is also considered, leading to a magnitude-reduced control torque that maintains the correct direction with respect to the spacecraft body. Simulation results show that the method works well.

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