

# Passivity-based attitude control with input quantization

Xiaoyu Lang<sup>1,2</sup>, Christopher J Damaren<sup>2</sup> and Xibin Cao<sup>1</sup>

Proc IMechE Part G:  
J Aerospace Engineering  
0(0) 1–6  
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sagepub.co.uk/journalsPermissions.nav  
DOI: 10.1177/0954410017754018  
journals.sagepub.com/home/pig



## Abstract

A passivity-based controller with quantization for spacecraft attitude control is developed. This passive control scheme includes two parts which are a proportional controller for quaternion feedback and a strictly positive real controller for the angular velocity. To alleviate the errors caused by quantization, a special modification for the nonlinear quantized input is employed in the strictly positive real controller. Asymptotic stability can be guaranteed with the presented controller structure. A guideline for the controller parameter selection is provided with sensitivity analysis for the control scheme. Numerical simulation results demonstrate the effectiveness of the proposed controller.

## Keywords

Passivity, spacecraft, quantization, attitude control, nonlinear modification

Date received: 17 July 2017; accepted: 18 December 2017

## Introduction

Wireless communication technology applied to spacecraft has become an attractive research area recently.<sup>1,2</sup> A typical application of this technology in space is the use of commercial-off-the-shelf (COTS) components in the assorted subsystems inside of spacecraft, as adopting the COTS components can reduce the cost of space missions.<sup>3</sup> Some practical space missions such as distributed spacecraft systems and repurposing the components of retired spacecraft can also employ the wireless link technology.<sup>4,5</sup> In these applications, the data transmission cannot be performed with infinite precision given the low-cost wireless components onboard spacecraft. As a crucial part of a spacecraft system, the stability of the attitude control subsystem is inherently tied with the success of space missions. Quantization is a viable alternative for the limited rate of data transmission between the attitude controller and spacecraft.

Considering the chaotic behavior exhibited from quantized-control in linear time-invariant (LTI) systems, some approaches, such as the sector bound method and quantization dependent Lyapunov functions, were presented to understand and mitigate the quantization errors from the control systems.<sup>6–8</sup> Motivated by the above research results, a robust time-varying sliding-mode controller was presented to stabilize a linear system with quantized measurements when saturation was considered.<sup>9</sup> Saturation in quantization was also discussed in a feedback control problem with the presence of time-delay nonlinearity.<sup>10</sup>

A policy adjusting the quantization parameters online was utilized to design a sliding-mode feedback control when the system has a dead-zone input.<sup>11</sup> Focusing on a class of nonlinear feedback control systems with input quantization, a logarithmic quantizer was employed in stabilization analysis to get a guideline on the parameter selection for the proposed adaptive back-stepping controller.<sup>12</sup> As an extension of the guideline derivation,<sup>12</sup> the quantization effects in spacecraft attitude control were addressed to examine the requirement on the moment of inertia about spacecraft in controller parameter design.<sup>13</sup> However, there is a little literature to treat quantization as an input nonlinearity on the attitude control for spacecraft.

It is well known that input nonlinearity inherently suffers from the presence of factors such as saturation, dead zone, relay, and quantization in the dynamic systems. A set of nonlinear modification for arbitrary input nonlinearities was developed to alleviate the degradation in performance from taking nonlinear input values.<sup>14</sup> This modification has then been applied to a Hammerstein system to design a passive nonlinear dynamic compensator.<sup>15</sup> In the case of

<sup>1</sup>Research Center of Satellite Technology, Harbin Institute of Technology, PR China

<sup>2</sup>Institute for Aerospace Studies, University of Toronto, Canada

### Corresponding author:

Xiaoyu Lang, Research Center of Satellite Technology, Harbin Institute of Technology, P. O. Box 3012, Harbin 150080, PR China.  
Email: xylang@hit.edu.cn

attitude control for spacecraft, the input torque with the nonlinear modification for saturation was developed with a passive angular velocity controller.<sup>16</sup> That passive control approach has been provided to enhance the robustness such as rejecting the modeling errors in spacecraft attitude control.<sup>17,18</sup> This special nonlinear modification with passive control is still applicable to relieve the effects from the quantized input of the attitude control and simultaneously ensures the stability of spacecraft dynamics.

In this paper, the major contribution consists of a special nonlinear controller input modification to build the feedback controller, which can compensate for the error from quantization and simultaneously ensure asymptotic stability around the equilibrium points. A strictly positive real (SPR) controller is proposed as a part of the feedback controller for the angular velocity. The proportional control for the quaternions serves as another part of the feedback controller. The proposed controller has no requirement for knowledge about the properties of the spacecraft like the moment of inertia, which can guarantee the robustness of the spacecraft dynamics. The numerical simulation results are presented at the end of this paper in two parts to assess the performance of the proposed controller. The first part is the sensitivity analysis of the parameters in the controller, which can provide a guideline for the parameter selection. In the second part, the results validate the effectiveness of the proposed passivity-based controller.

## Spacecraft attitude dynamics

The equation governing the attitude dynamics of a rigid-body spacecraft is the classical Euler equation<sup>19</sup>

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^\times \mathbf{I}\boldsymbol{\omega} = \boldsymbol{\tau} \quad (1)$$

where the moment of inertia matrix is  $\mathbf{I} \in \mathbb{R}^{3 \times 3}$  and  $\boldsymbol{\tau} \in \mathbb{R}^3$  represents the control torque input. The angular velocity  $\boldsymbol{\omega} \in \mathbb{R}^3$  is expressed in the body frame. The operator  $(\cdot)^\times$  is the skew-symmetric matrix for a vector  $\mathbf{a} = [a_1 \ a_2 \ a_3]^\top$  which is given by

$$\mathbf{a}^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

It has the property  $(\mathbf{a}^\times)^\top = -\mathbf{a}^\times$ . The kinematic equation describing the evolution of the quaternions  $\{\boldsymbol{\epsilon}, \beta\}$  is

$$\begin{bmatrix} \dot{\boldsymbol{\epsilon}} \\ \dot{\beta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \boldsymbol{\eta} \mathbf{1} + \boldsymbol{\epsilon}^\times \\ -\boldsymbol{\epsilon}^\top \end{bmatrix} \boldsymbol{\omega} \quad (2)$$

The quaternions are denoted by  $\boldsymbol{\eta} \in \mathbb{R}$  and  $\boldsymbol{\epsilon} \in \mathbb{R}^3$  which also satisfy  $\boldsymbol{\epsilon}^\top \boldsymbol{\epsilon} + \boldsymbol{\eta}^2 = 1$ .

## Quantization

This paper considers a logarithmic quantizer  $\mathcal{Q}(\cdot)$ .<sup>7,8</sup> It has the following characteristic

$$\mathcal{U} = \{\pm u_i : u_i = \rho^{1-i} u_0, i = \pm 1, \pm 2, \pm 3 \dots\} \cup \{\pm u_0\} \cup \{0\}, u_0 > 0 \quad (3)$$

where  $\mathcal{U}$  denotes the quantization levels and

$$u_0 = u_{\min}(1 + \delta), \quad \delta = \frac{1 - \rho}{1 + \rho}, \quad 0 < \rho < 1 \quad (4)$$

The parameter  $u_{\min}$  determines the size of the dead-zone for the quantizer  $\mathcal{Q}(\cdot)$ . The quantization density  $\rho$  is defined as in Fu and Xie<sup>7</sup> and Gao and Chen.<sup>8</sup> The traditional definition of quantization density is represented by  $\eta_f$ , and the relationship between  $\rho$  and  $\eta_f$  is  $\eta_f = -2/\ln \rho$ . This is the reason why this quantization approach is named the logarithmic quantizer.

According to these definitions, the quantization function can be given in an equivalent form as follows

$$\mathcal{Q}(s) = \begin{cases} u_i, & \frac{1}{1+\delta} u_i < s \leq \frac{1}{1-\delta} u_i, \\ & (i = 1, 2, 3, \dots) \\ 0, & 0 \leq s \leq \frac{1}{1+\delta} u_0 \\ -\mathcal{Q}(-s), & s < 0 \end{cases} \quad (5)$$

The quantization function  $\mathcal{Q}(\cdot)$  is illustrated in Figure 1.

## Controller design

The proposed feedback regulator is given by

$$\boldsymbol{\tau}(t) = -\mathbf{u}_p - \mathcal{Q}(\mathbf{y}_c(t)) \quad (6)$$

where  $\mathbf{u}_p = k\boldsymbol{\epsilon}$ ,  $k > 0$  is the proportional control for the quaternions and  $\mathcal{Q}(\mathbf{y}_c)$  is the output of the quantizer. The quantization controller diagram is illustrated in Figure 2, where we see that  $\mathbf{y}_c$  is the output of a SPR controller. It should be emphasized that it is the torque output of the SPR controller that is quantized. The measurements  $\boldsymbol{\omega}$  and  $\boldsymbol{\epsilon}$  are not quantized.

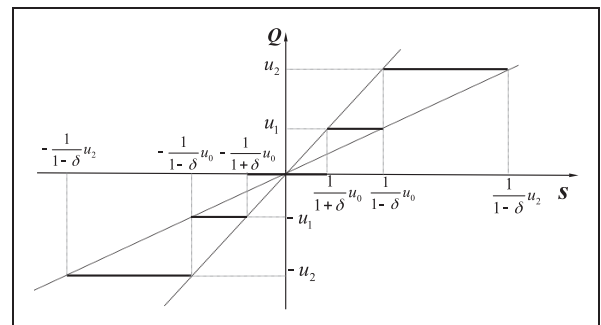
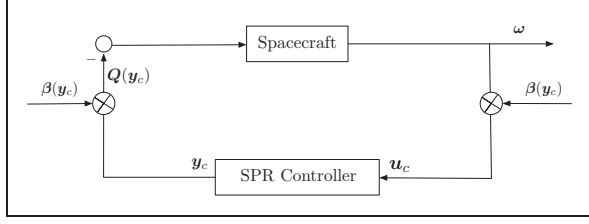


Figure 1. Quantization.



**Figure 2.** Quantization controller.

This paper will utilize the passivity property of the mapping from  $\mathbf{Q}(y_c)$  to  $\omega$  to propose a control scheme. It is shown that a SPR controller interconnected with a passive system provides input–output stability.<sup>20</sup> A significant lemma on passivity of LTI systems is given by:<sup>21</sup>

*Lemma 1:* Kalman–Yakubovich–Popov (KYP) Lemma.

Consider a system given in the form of the state-space equations

$$\dot{\mathbf{x}}_c = \mathbf{A}_c \mathbf{x}_c + \mathbf{B}_c \mathbf{u}_c, \mathbf{y}_c = \mathbf{C}_c \mathbf{x}_c \quad (7)$$

where the matrices  $\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c$  form a minimal state-space realization. Assuming that  $\mathbf{A}_c$  is Hurwitz, this system is SPR if and only if there are real matrices  $\mathbf{P}_c = \mathbf{P}_c^T > 0$  and  $\mathbf{Q}_c = \mathbf{Q}_c^T > 0$  which satisfy the conditions

$$\begin{aligned} \mathbf{P}_c \mathbf{A}_c + \mathbf{A}_c^T \mathbf{P}_c &= -\mathbf{Q}_c \\ \mathbf{P}_c \mathbf{B}_c &= \mathbf{C}_c^T \end{aligned} \quad (8)$$

For a proof, see Wen<sup>21</sup>

Motivated by Bernstein and Haddad<sup>14</sup> and Haddad and Chellaboina,<sup>15</sup> the nonlinear modification  $\beta$  is defined as  $\beta(y_c)y_c = \mathbf{Q}(y_c)$  and  $\beta(y_c) = \text{diag}\{\beta_1(y_{c1}), \beta_2(y_{c2}), \beta_3(y_{c3})\}$  with

$$\beta_i(y_{ci}) = \begin{cases} \mathbf{Q}_i(y_{ci})/y_{ci}, & y_{ci} \neq 0 \\ 1 & y_{ci} = 0 \end{cases} \quad (9)$$

The signal  $y_c$  is the output of the SPR control system as given by equation (7) and the input of the SPR control system is modified by  $\beta$  using  $\mathbf{u}_c = \beta(y_c)\omega$ .

A theorem is now given to demonstrate the stability of the control scheme.

*Theorem 1:* Consider the system given by equations (1) and (2), the controller shown in equations (6), (7), (9) and  $\mathbf{u}_c = \beta(y_c)\omega$  guarantees that the closed-loop system is asymptotically stable with respect to the equilibrium point  $(\epsilon, \omega, \mathbf{x}_c) = (\mathbf{0}, \mathbf{0}, \mathbf{0})$ .

*Proof:* For the proof of stability, we choose the Lyapunov candidate

$$V = \frac{1}{2} \omega^T \mathbf{I} \omega + k[\epsilon^T \epsilon + (\eta - 1)^2] + \frac{1}{2} \mathbf{x}_c^T \mathbf{P}_c \mathbf{x}_c \quad (10)$$

Taking the time derivative of the Lyapunov function gives

$$\begin{aligned} \dot{V} &= \omega^T \mathbf{I} \dot{\omega} + k\epsilon^T \dot{\epsilon} + \frac{1}{2} \mathbf{x}_c^T (\mathbf{P}_c \mathbf{A}_c + \mathbf{A}_c^T \mathbf{P}_c) \mathbf{x}_c + \mathbf{x}_c^T \mathbf{P}_c \mathbf{B}_c \mathbf{u}_c \\ &= \omega^T (-k\epsilon - \mathbf{Q}(y_c)) + k\epsilon^T \omega - \frac{1}{2} \mathbf{x}_c^T \mathbf{Q}_c \mathbf{x}_c \\ &\quad + \mathbf{x}_c^T \mathbf{C}_c^T \beta(y_c) \omega \\ &= -\mathbf{y}^T \beta(y_c) y_c + [\beta(y_c) y_c]^T \mathbf{y} - \frac{1}{2} \mathbf{x}_c^T \mathbf{Q}_c \mathbf{x}_c \\ &= -\frac{1}{2} \mathbf{x}_c^T \mathbf{Q}_c \mathbf{x}_c \leq 0 \end{aligned} \quad (11)$$

Examining  $\dot{V} = 0$  implies that  $\mathbf{x}_c = \mathbf{0}$  since  $\mathbf{Q}_c = \mathbf{Q}_c^T > 0$ . This implies that  $\dot{\mathbf{x}}_c = \mathbf{y}_c = \mathbf{0}$ . From equation (7), we have  $\mathbf{B}_c \beta \omega = \mathbf{0}$  and  $\omega = 0$  follows since  $\mathbf{B}_c$  has full column rank. According to equation (2),  $\epsilon = \mathbf{0}$ . Therefore, we can find the largest invariant set  $\mathbb{M} = \{\epsilon = \mathbf{0}, \omega = \mathbf{0}, \mathbf{x}_c = \mathbf{0}\}$  which forces the state of the system to asymptotically approach the set  $\mathbb{M}$  based on Lasalle's invariance principle in Marquez.<sup>20</sup> Thus,  $(\epsilon, \omega, \mathbf{x}_c) \rightarrow (\mathbf{0}, \mathbf{0}, \mathbf{0})$  is asymptotically stable.

Now we use the KYP lemma to design a LTI controller. In particular, the linear quadratic regulator (LQR) is employed to design an SPR controller using the method in Benhabib et al.<sup>22</sup> With the small angle and rate assumption,  $\omega \doteq \dot{\theta}$ , the spacecraft dynamic system can be linearized as follows

$$\begin{aligned} \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} &= \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{I}^{-1}k & \mathbf{0} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{I}^{-1} \end{bmatrix}}_{\mathbf{B}} \mathbf{Q}(u), \\ \mathbf{y} &= \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{1} \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \end{aligned} \quad (12)$$

where  $\mathbf{1}$  is the identity matrix. According to the algorithm,<sup>22</sup> the matrix  $\mathbf{C}_c$  in the SPR controller in equation (8) can be designed as a state-feedback gain using the LQR standard formula by choosing weight matrices  $\mathbf{Q}_{lqr} = \mathbf{Q}_{lqr}^T \geq 0$  and  $\mathbf{R} = \mathbf{R}^T > 0$ . After that, a Hurwitz matrix  $\mathbf{A}_c = \mathbf{A} - \mathbf{B}\mathbf{C}_c$  is formulated. Then, we can determine  $\mathbf{P}_c$  by selecting a suitable  $\mathbf{Q}_c$  in equation (8). We take  $\mathbf{B}_c = \mathbf{P}_c^{-1} \mathbf{C}_c^T$ . This yields a SPR controller for the angular velocity.

## Numerical simulation

In this section, a numerical simulation of the proposed controller is given. The initial values for simulation are set to  $\eta_0 = 0.9470$ ,  $\epsilon_0 = [-0.1500, 0.2805, -0.0445]^T$ ,  $\omega_0 = [0, 0, 0]^T$ , and  $\mathbf{x}_c = [-0.014, 0.0716 - 0.0314, 0, 0, 0]$ . The moment of inertia used in the example is

$$\mathbf{I} = \begin{bmatrix} 147 & 6.5 & 6 \\ 6.5 & 158 & 5.5 \\ 6 & 5.5 & 137 \end{bmatrix} \text{kg} \cdot \text{m}^2$$

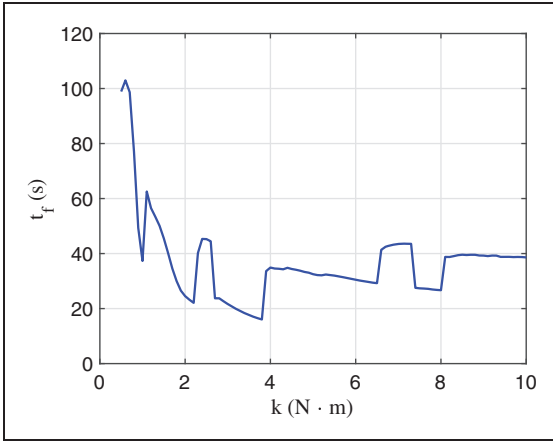


Figure 3. Effects from gain  $k$  to attitude convergence time.

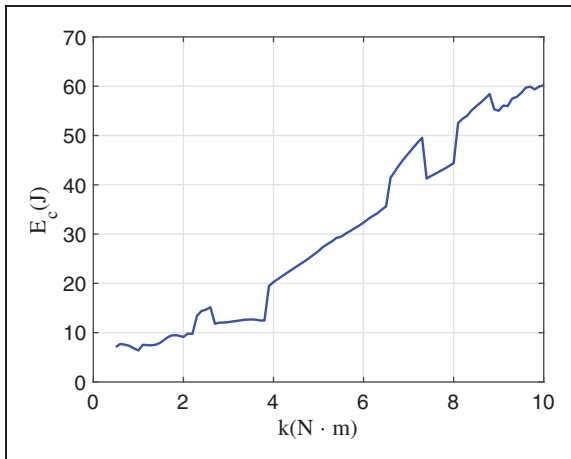


Figure 4. Effects from gain  $k$  to consumed energy.

The passivity-based control scheme proposed in this paper presents no requirements on the quantization parameters. We present a sensitivity analysis of those parameters which can affect the performance. In this work, convergence time and consumed energy are examined. Convergence time,  $t_f$ , is defined as the time that the quaternion  $\epsilon$  reaches and remains in a certain band from the initial values as  $\|\epsilon(t)\| \leq 0.02 \|\epsilon(0) - \epsilon(\infty)\|$ , ( $t \geq t_f$ ). The consumed energy is defined as

$$E_c = \int_{t_0}^{t_f} \|\mathbf{u}\|_1 dt \quad (13)$$

At first, we choose the gain  $k$  of the controller based on the sensitivity analysis in Figures 3 and 4. From Figure 3, it is clear that the attitude convergence time can be kept below 30s for some values of the gain  $k$  in the range 0, 10. The energy consumption increases with gain in Figure 4. We choose  $k = 1.76$  as the value of gain for the following simulation, based on the consideration of attitude convergence time and consumed energy. The two quantization parameters  $\delta$  and  $u_{\min}$  are varied so as to analyze their effects on the

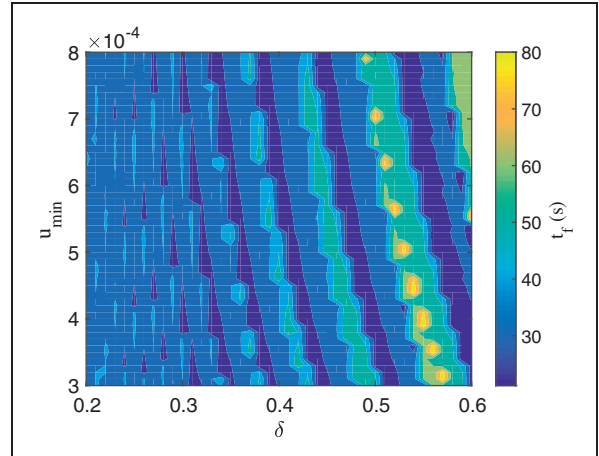


Figure 5. Effects from parameters in quantization to attitude convergence time.

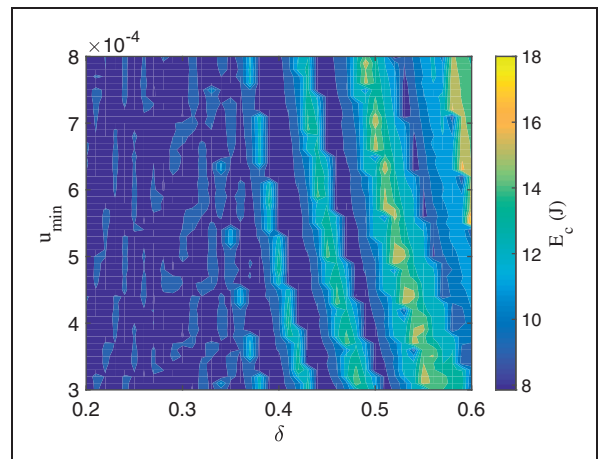


Figure 6. Effects from parameters in quantization to consumed energy.

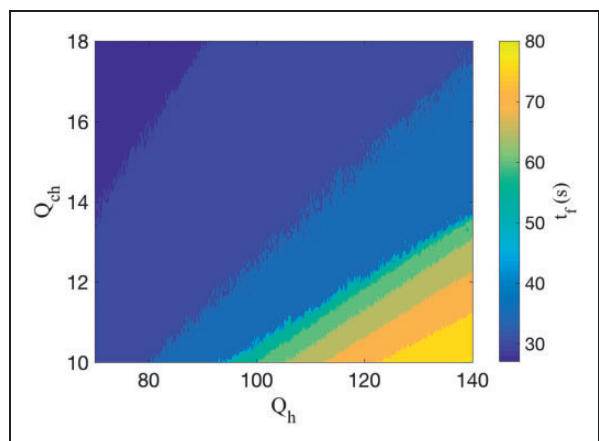
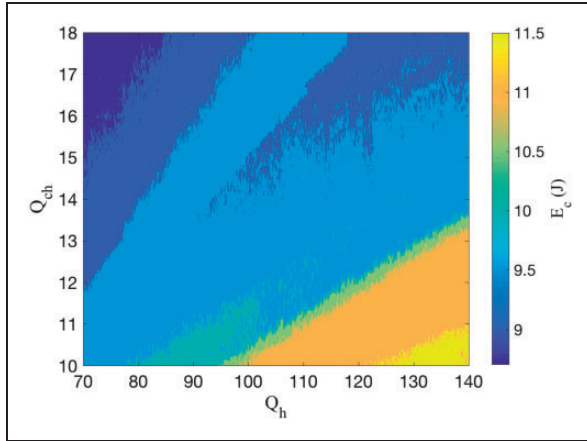
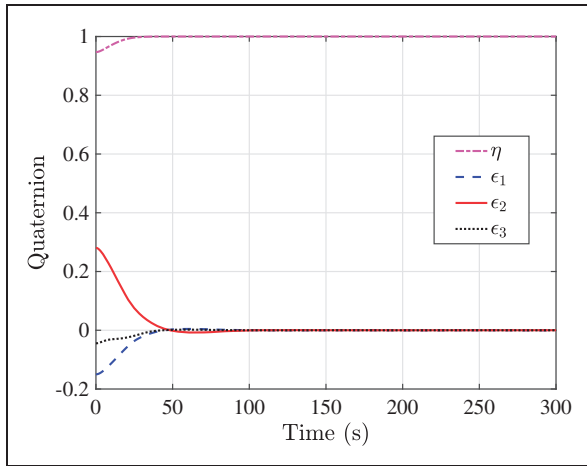


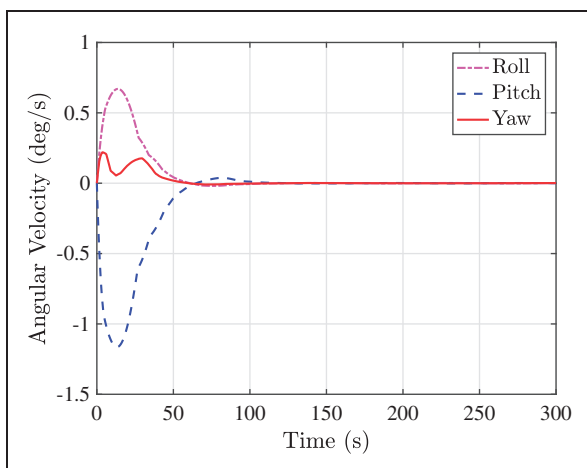
Figure 7. Effects from parameters in SPR to attitude convergence time. SPR: strictly positive real.



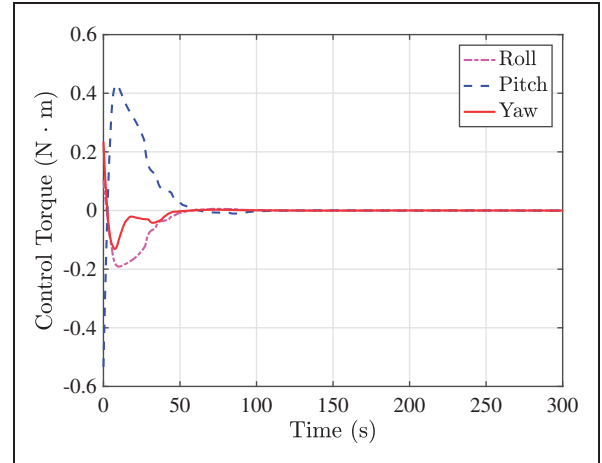
**Figure 8.** Effects from parameters in SPR to consumed energy. SPR: strictly positive real.



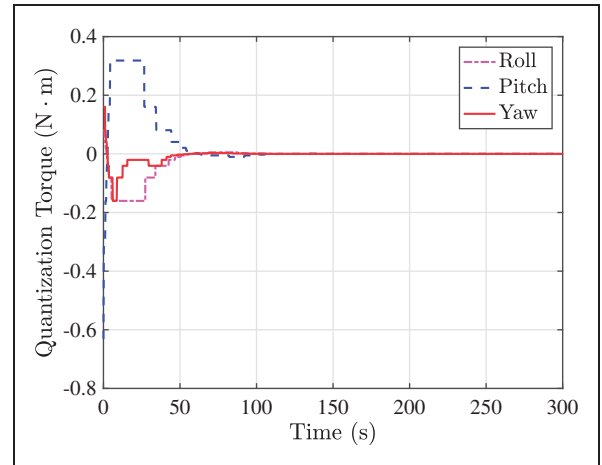
**Figure 9.** Quaternion vs. time.



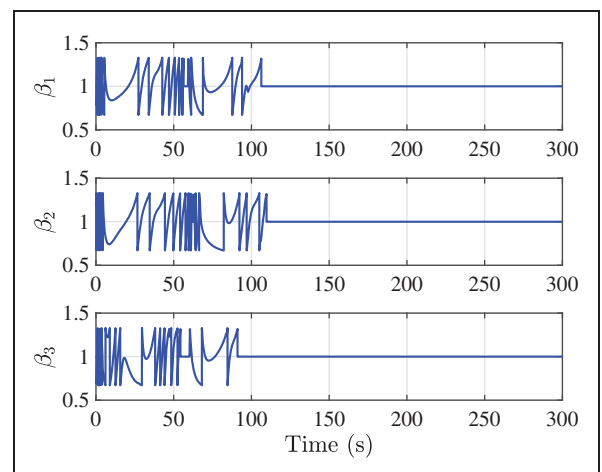
**Figure 10.** Angular velocity vs. time.



**Figure 11.** Control torque.



**Figure 12.** Quantization Torque.



**Figure 13.**  $\beta$  vs. time.

attitude convergence time and energy consumption in Figures 5 and 6.

From Figures 5 and 6, the dark areas represent good performance in the convergence time of the

attitude and the energy consumption. The dark areas in the two figures correspond to the range of  $\delta$  and  $u_{\min}$  as  $[0.2, 0.6]$  and  $[0.0003, 0.0008]$ , respectively. We choose  $\delta = 0.33$ ,  $u_{\min} = 0.0005$  as the values of the quantization for the final simulation.



One of the contributions of this paper is to show that introducing an SPR controller can eliminate the negative effects of the quantized input to the controller. We choose parameters  $Q_h$ ,  $Q_{ch}$ , and  $R_h$  in the LQR algorithm as follows:  $Q_{lqr} = Q_h \cdot \text{diag}\{1, 1, 1, 10, 10, 10\}$ ,  $Q_c = Q_{ch} \cdot \text{diag}\{1, 1, 1, 10, 10, 10\}$ , and  $R = \text{diag}\{1.1, 1.1, 1.1\}$ . The sensitivity plots for these three parameters are shown in Figures 7 and 8.

The dark areas in Figures 7 and 8 correspond to smaller attitude convergence time and smaller consumed energy. Hence, we select  $Q_{lqr} = \text{diag}\{90, 90, 90, 900, 900, 900\}$  and  $Q_c = \text{diag}\{16, 16, 16, 160, 160, 160\}$  as values for the simulation.

With the help of the sensitivity analysis, the resulting behaviors are illustrated in Figures 9 to 13. It is easily seen from these figures that the proposed passivity-based controller with quantization can ensure good performance. The stability of the control scheme is verified as well.

## Conclusion

A passivity-based attitude control scheme for spacecraft has been proposed to mitigate the effects caused by quantization. This feedback controller consists of two parts, a proportional controller for the quaternions and a SPR controller for the angular velocity. A special modification for the controller input is utilized to alleviate the errors caused by the quantization. The sensitivity analysis is given which provides a guideline for selecting the controller parameters. Based on this selection, the simulation results are given to verify the performance of the proposed passive controller.

## Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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