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Comments on "Fully magnetic attitude control for spacecraft subject to gravity gradient"☆

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Abstract

The Lyapunov argument used in Wiśniewski and Blanke, (Automatica 35 (1999) 1201) to establish asymptotic stability of a magnetic control law for an earth-orbiting spacecraft is incorrect. It is shown here that a small change to the assumed form of the control law can remedy the situation.

Keywords: Spacecraft magnetic control

1. Error and correction

Wiśniewski and Blanke (1999) adopt the following model for a rigid spacecraft subject to the gravity-gradient torque:

$$\mathbf{I}^c\dot{\mathbf{\Omega}}_{cw}(t) = -{}^c\mathbf{\Omega}_{cw}(t) \times \mathbf{I}^c\mathbf{\Omega}_{cw}(t) + {}^c\mathbf{N}_{ctrl}(t) + {}^c\mathbf{N}_{gg}(t), \quad (1)$$

where $\mathbf{I}$ is the moment of inertia matrix, ${}^c\mathbf{\Omega}_{cw}$ is the spacecraft angular velocity with respect to an inertial (world) coordinate system, and ${}^c\mathbf{N}_{ctrl}$ and ${}^c\mathbf{N}_{gg}$ are, respectively, the control and gravity-gradient torques. It is assumed that the control torque is generated by a magnetic moment ${}^c\mathbf{m}$ in the presence of the earth's magnetic field ${}^c\mathbf{B}$ according to

$$ {}^c\mathbf{N}_{ctrl} = {}^c\mathbf{m} \times {}^c\mathbf{B}. \quad (2)$$

They adopt the system total energy E_{tot} as a Lyapunov function and correctly show that

$$\dot{E}_{tot} = {}^c\mathbf{\Omega}_{co}^T\,{}^c\mathbf{N}_{ctrl}, \quad (3)$$

where ${}^c\mathbf{\Omega}_{co}$ is the angular velocity of the spacecraft with respect to an orbital frame. Their assumed form for the magnetic moment feedback law is

$$ {}^c\mathbf{m} = (\mathbf{H}^c\mathbf{\Omega}_{co}) \times {}^c\mathbf{B}, \quad (4)$$

where $\mathbf{H}$ is a positive definite gain matrix. This leads to

$$\dot{E}_{tot} = -{}^c\mathbf{\Omega}_{co}^T\tilde{\mathbf{B}}^T\tilde{\mathbf{B}}\mathbf{H}^c\mathbf{\Omega}_{co}, \quad (5)$$

where $\tilde{\mathbf{B}}$ is the skew-symmetric matrix used to implement the cross product. Wiśniewski and Blanke conclude that since $\tilde{\mathbf{B}}^T\tilde{\mathbf{B}}$ is positive semidefinite and $\mathbf{H}$ is positive definite, then $\dot{E}_{tot}$ is negative semidefinite. Clearly, this is false since the product of a positive definite matrix and a positive semidefinite one does not, in general, possess any definiteness property. The problem can be fixed by changing the control law in Eq. (4) to

$$ {}^c\mathbf{m} = \mathbf{H}({}^c\mathbf{\Omega}_{co} \times {}^c\mathbf{B}), \quad (6)$$

which, using Eqs. (2), (3), and (6) leads to

$$\dot{E}_{tot} = -{}^c\mathbf{\Omega}_{co}^T\tilde{\mathbf{B}}^T\mathbf{H}\tilde{\mathbf{B}}{}^c\mathbf{\Omega}_{co}.$$

Since $\tilde{\mathbf{B}}^T\mathbf{H}\tilde{\mathbf{B}}$ is positive semidefinite, one now clearly has $\dot{E}_{tot} \leqslant 0$ and one can go on to apply the Krasovskii–LaSalle lemma as the authors have done. However, the invariant set that they have used (${}^c\mathbf{B} \times {}^c\mathbf{\Omega}_{co} = \mathbf{0}$ or ${}^c\mathbf{B} \times (\mathbf{H}^c\mathbf{\Omega}_{co}) = \mathbf{0}$) must be amended to remove the second class of angular velocities.

References

Wiśniewski, R., & Blanke, M. (1999). Fully magnetic attitude control for spacecraft subject to gravity gradient. *Automatica*, *35*, 1201–1214.

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